1.2. DIFFERENTIAL EQUATION OF FIRST ORDER AND FIRST DEGREE

1.2.1. Introduction

The differential equation of the first order and first degree is the simplest type of differential equation. It is represented as:

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \quad .....(1)$$

For solving $\frac{dy}{dx}$ we can write equation (1) as:

$$\frac{dy}{dx} = f(x, y) \quad .....(2)$$

and if we write $f(x, y) = -\frac{M(x, y)}{N(x, y)}$

Then equation (2) can be placed in the form of

$$Mdx + Ndy = 0 \quad .....(3)$$

Where, M and N are the functions of x and y.

1.2.2. Geometrical Meaning of Differential Equation of First Order and First Degree

A first order and first degree differential equation involves the independent variable x (say), dependent variable y (say) so, it can be put in any one of the following forms:

$\frac{dy}{dx} = f(x, y)$ or $f(x, y) = 0$, or

$$f(x, y)\, dx + g(x, y)dy = 0 \quad ......(1)$$

where $\frac{dy}{dx}$ represents the slope of the tangent to the curve of the function y(x) and from equation (1) has a solution y(x) passing through a point $A_1(x_1, y_1)$ of the xy-plane.

We obtained the value of $\frac{dy}{dx}$ at $A_1(x_1, y_1)$ after substituting the coordinates of the point $A_1(x_1,y_1)$ in equation (1) which is represented by m_1. Thus m_1 is the slope of the tangent to the curve at $A_1(x_1,y_1)$.

From equation (1) a point that moves through $A_1(x_1, y_1)$ subject to the restriction imposed must move in the direction m_1.

Suppose that the point $A_2(x_2, y_2)$ goes to the m_1 direction, for the shortest distance from the point $A_1(x_1, y_1)$.

Assume that the slope of the tangent is m_2 at $A_2(x_2, y_2)$ and the point move in the direction of m_2.

Suppose that the point moves to the direction m_2 by a short distance and arrive at the position $A_3(x_3, y_3)$.

Using this method, let's say the point one goes to the point $A_4(x_4, y_4)$ and so on to the successive points. Therefore, a curve will pass through the point $A_1(x_1, y_1)$.

Correspondingly, a curve passes through every point on the xy-plane, the coordinates of each point and the direction of the tangent to it will satisfy differential equation (1).

Therefore equation (1) denotes a family of curves like that each point in the xy-plane passes through one of the family of curves.

Example 28: Give geometrical meaning of the solution of the differential equation $\frac{d^2y}{dx^2} = 0$.

Solution: The differential equation is,

$$\frac{d^2y}{dx^2} = 0 \qquad ...(1)$$

On Integration of equation (1) we have

$\frac{dy}{dx} = m$ where m is an arbitrary constant.

Again integrating, we have $y = mx + c$ ...(2)

in equation (2) c is another arbitrary constant.

This equation (2) is the general solution of differential equation (1).

Now a line through any point (0,c) drawn in any direction m is the locus of a particular integral of Equation (1).

There will be infinity of lines corresponding to the infinity of the values of m if we assign a particular value of c_1 to c and all these lines will be loci of integrals.

Because of the infinity of values that can be given to m, each of the infinity of values that can be given to c, there corresponds an infinity of lines.

Therefore, the general solution of the equation represents a double infinite system of lines. Similarly the locus of the differential equation (1) consists of a double infinite system of lines.

Example 29: Demonstrate that the curves that have the angle between the tangent and the radius vector half point of the vectorial angle is cardioid.

Solution: We have, $\phi = \frac{1}{2}\theta$, θ stands for vectorial angle.

$\therefore \tan\phi = \tan\frac{1}{2}\theta$, but $\tan\phi = r(d\theta/dr)$.

$\therefore r(d\theta/dr) = \tan\frac{1}{2}\theta$, or $(dr/r) = \left(\cot\frac{1}{2}\theta\right)d\theta$.